

Chapter 13

STATIC ELECTRIC FORCES AND FIELDS

Note: For those interested in a review of atomic bonding, charge characteristics, and the characteristics of conductors and insulators, see the addendum at the end of the chapter.

A.) Coulomb's Law:

1.) From experimental observation, Charles Coulomb found that the magnitude of the electrical force F_c acting upon a point charge q_1 due to the presence of a second point charge q_2 a distance r meters away is:

$$F_c = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2},$$

where ϵ_0 is called the permittivity of free space (8.85×10^{-12} Farads/meter) and the $\frac{1}{4\pi\epsilon_0}$ term is a proportionality constant numerically equal to 9×10^9 nt·m²/C²

in the MKS system (this term is sometimes defined as $k = \frac{1}{4\pi\epsilon_0}$ to save space).

The equation is called Coulomb's Law.

Note 1: Historically, the symbol for charge has always been q or Q . In the MKS system, charge has the units of coulombs (symbol C).

Note 2: The charge on an electron is called the elementary charge unit and is given the symbol e . In the MKS system, it is equal to 1.6×10^{-19} coulombs/electron. Inverting that number gives us the number of electrons per coulomb. Doing the inversion shows that it takes 6.25×10^{18} electrons to generate a one coulomb charge.

Note 3: It is not uncommon to see charge values represented as, say, $12 \mu\text{C}$. The symbol μ stands for micro and is equal to 10^{-6} . When using Coulomb's Law, charge values must be in terms of coulombs, never micro-coulombs (even if the charge value is given in micro-coulombs in a problem).

Note 4: It has been experimentally observed that two like charges (i.e., two positive charges or two negative charges) will repulse one another while two unlike charges (a positive and a negative) will be attracted to one another. This is going to be useful in determining the direction of a Coulomb force on a given charge.

2.) Coulomb's Law is useful for determining the magnitude of the force between two point charges. To determine the direction, one must eyeball the situation.

a.) Example #1: Charges $q_1 = 2 \mu\text{C}$ and $q_2 = -3 \mu\text{C}$ are 10 centimeters apart. What is the force $F_{1,2}$ that charge q_1 exerts on charge q_2 ?

i.) Using Coulomb's Law to determine the force magnitude:

$$\begin{aligned} F_{1,2} &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \\ &= (9 \times 10^9 \text{ nt} \cdot \text{m}^2 / \text{C}^2) \frac{(2 \times 10^{-6} \text{ C})(3 \times 10^{-6} \text{ C})}{(.1 \text{ m})^2} \\ &= 5.4 \text{ nts.} \end{aligned}$$

Note: In the above equation, only the magnitude of charge q_2 was used. If we had included the charge's negative sign, our force calculation would have yielded a negative magnitude. Aside from the fact that magnitudes are supposed to be positive, a negative sign in front of the force equation could be construed as denoting direction. Though the two charges will always attract, the force direction depends solely upon where q_2 is, relative to q_1 .

directional possibilities for the force on charge q_2 due to the presence of charge q_1

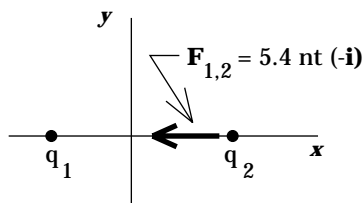


FIGURE 13.1a

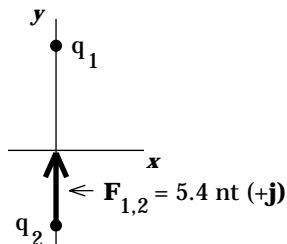


FIGURE 13.1b

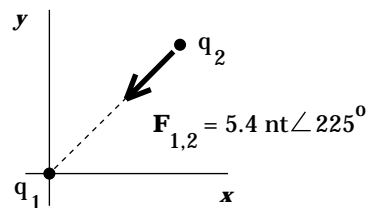


FIGURE 13.1c

ii.) To determine direction, we need more information than was given above. We know that the force is an attractive one (the charges are unlike), but we do not know how the charges are oriented relative to one another and to the coordinate axis being used in the problem. Figures 13.1a, b, and c on the previous page show three possibilities complete with the final force representation for each case.

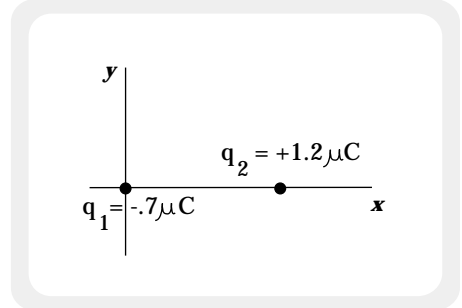


FIGURE 13.2

3.) Example #2: A charge $q_1 = -0.7 \mu\text{C}$ is located at the origin. A charge $q_2 = 1.2 \mu\text{C}$ is located $x = .6$ meters down the x axis as shown in Figure 13.2. Both charges are fixed to the axis. Where on the x axis might we put a third charge $q_3 = -2 \mu\text{C}$ so that the net force on q_3 is zero?

a.) Consider the direction of the forces acting on q_3 assuming q_3 is put:

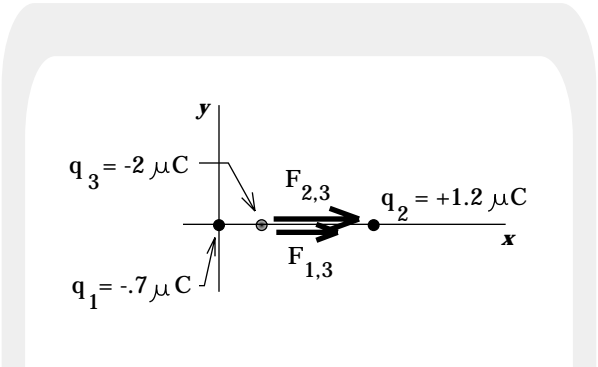


FIGURE 13.3a

i.) Between the two fixed charges (see sketch in Figure 13.3a). In that case, q_1 repulses q_3 pushing it to the right; q_2 attracts q_3 pulling it to the right.

At no position between the two fixed charges will the net force on q_3 add to zero.

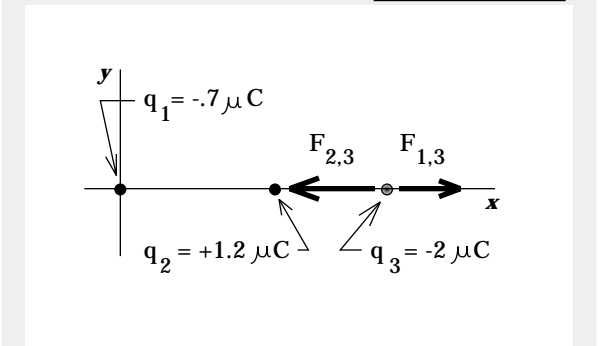


FIGURE 13.3b

ii.) To the right of both the fixed charges (see sketch in Figure 13.3b): q_1 repulses q_3 pushing it to the right; q_2 attracts q_3 pulling it to the left.

The forces will be in opposite directions, but because the smaller charge q_1 is always farther away from q_3 ,

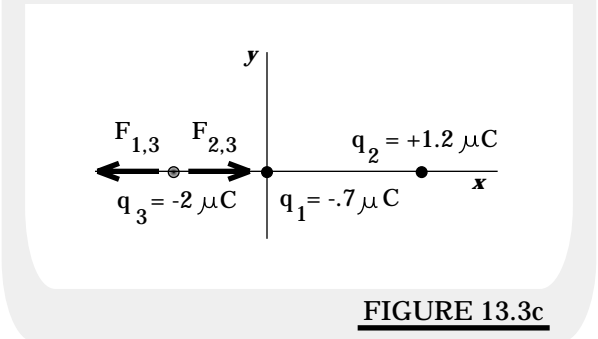


FIGURE 13.3c

there will never be a position to the right of the two fixed charges where the net force on q_3 adds to zero.

iii.) To the left of both the fixed charges (see sketch in Figure 13.3c on previous page): q_1 repulses q_3 pushing it to the left; q_2 attracts q_3 pulling it to the right.

The forces will be in opposite directions and the $1.2 \mu\text{C}$ charge (q_2) will be further away from q_3 than q_1 . As such, the smaller charge will be able to overcome the attraction of the larger charge which means there will be some position where the two exactly balance one another out.

b.) Having decided to put q_3 to the left of the two fixed charges, we will assign a coordinate x to q_3 and proceed to determine the net force acting on q_3 due to the other charges. This net force must equal zero (that was the question: where will the net force on q_3 equal zero?). Eyeballing the force directions and using N.S.L. on q_3 , we get:

$$\begin{aligned} \underline{\Sigma F_x}: \\ -F_{1,3} + F_{2,3} &= 0 \\ \Rightarrow \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{x^2} &= \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{(x + .6)^2} \\ \Rightarrow \frac{q_1}{x^2} &= \frac{q_2}{(x + .6)^2}. \end{aligned}$$

From here, the problem is simple algebra.

4.) Example #3: Consider a fixed charge $-Q_1$ located a distance a units up the y -axis, a second fixed charge Q_2 located a distance b units down the y axis (i.e., at $y = -b$), and a third charge $-Q$ placed an arbitrary distance c units down the x axis (see Figure 13.4). What is the net force acting on $-Q$ due to the presence of $-Q_1$ and Q_2 ?

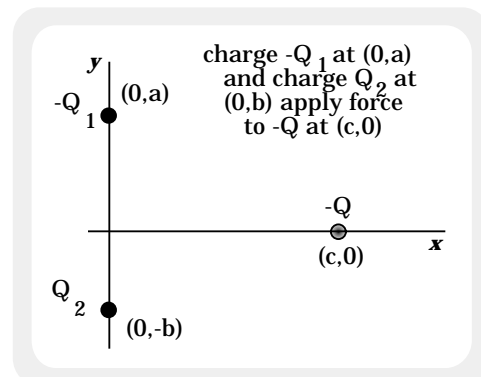


FIGURE 13.4

a.) To begin with, the answer is NOT $kQ_1 Q/r_1^2 - kQ_2 Q/r_2^2$ or some

such linear sum. Forces are not scalars--they have to be added VECTORIALLY.

b.) Keeping that in mind, we need to determine:

i.) The direction of each force acting on -Q;

ii.) The magnitude of each force acting on -Q;

iii.) The x and y components of each force acting on -Q, and;

iv.) The vector sum of those components.

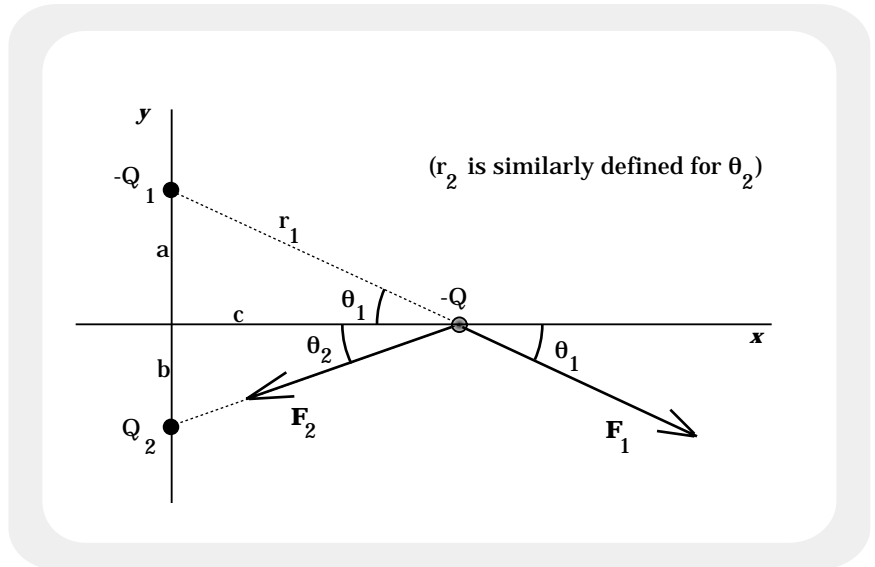


FIGURE 13.5a

c.) The direction of force on -Q due to the presence of -Q₁ will be along a line between -Q and -Q₁. The charges are like-charges so the force will be repulsive and the force direction will be away from -Q₁.

The direction of force on -Q due to the presence of Q₂ will be along a line between -Q and Q₂. The charges are unlike-charges so the force will be attractive and the force direction will be toward Q₂. The results of these observations are shown in Figure 13.5a.

d.) The magnitude of each force is determined by Coulomb's Law:

$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q}{r_1^2} \quad \text{and} \quad F_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_2 Q}{r_2^2},$$

where $r_1 = (a^2 + c^2)^{1/2}$ and $r_2 = (b^2 + c^2)^{1/2}$.

e.) To put the forces into component form (we must do that before we can add them vectorially), we need to determine the sine and cosine

of both θ_1 and θ_2 . As we haven't been given those angles, we need to use trickery to determine those sine and cosine quantities in terms of variables we know (i.e., terms like a, b, or c). Using the right triangles shown in Figure 13.5b, we find:

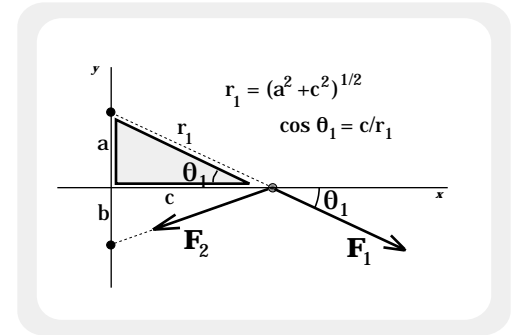


FIGURE 13.5b

i.) For $\sin \theta_1$:

$$\begin{aligned}\sin \theta_1 &= (\text{opp. side})/(\text{hyp.}) \\ &= a/(a^2 + c^2)^{1/2}.\end{aligned}$$

ii.) For $\cos \theta_1$:

$$\begin{aligned}\cos \theta_1 &= (\text{adj. side})/(\text{hyp.}) \\ &= c/(a^2 + c^2)^{1/2}.\end{aligned}$$

iii.) Similarly:

$$\begin{aligned}\sin \theta_2 &= b/(b^2 + c^2)^{1/2} \quad \text{and} \\ \cos \theta_2 &= c/(b^2 + c^2)^{1/2}.\end{aligned}$$

f.) We are now in a position to write vectorially the sum of the forces acting on $-Q$:

i.) Expanding the x component of force F_1 (i.e., writing the component as $F_1 \cos \theta_1$ using the information from Figure 13.5b):

$$\begin{aligned}F_{1,x} &= F_1 \cos \theta_1 \\ &= \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q}{(a^2 + c^2)^{1/2})^2} \left[\frac{c}{(a^2 + c^2)^{1/2}} \right] \\ &= \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q c}{(a^2 + c^2)^{3/2}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q c}{r_1^3}.\end{aligned}$$

Note: The defined value of r_1 was $r_1 = (a^2 + c^2)^{1/2}$. The variable r has been used above because things are about to get messy. Brevity here will help.

ii.) Similarly, the entire force equation becomes:

$$(F_1 \cos \theta_1 - F_2 \cos \theta_2) \mathbf{i} + (-F_1 \sin \theta_1 - F_2 \sin \theta_2) \mathbf{j}.$$

iii.) In expanded form, this becomes:

$$\mathbf{F} = \frac{Q}{4\pi\epsilon_0} \left[\left(\frac{Q_1 c}{(a^2 + c^2)^{3/2}} - \frac{Q_2 c}{(b^2 + c^2)^{3/2}} \right) \mathbf{i} + \left(-\frac{Q_1 a}{(a^2 + c^2)^{3/2}} - \frac{Q_2 b}{(b^2 + c^2)^{3/2}} \right) \mathbf{j} \right].$$

Big Note: This problem would have been considerably easier if there had been more symmetry. For instance, if the magnitude of $-Q_1$ had equaled the magnitude of Q_2 , and if the distance a had equaled the distance b , then both forces F_1 and F_2 would have had the same magnitude (i.e., $F_1 = F_2$) and angle (i.e., $\theta_1 = \theta_2$), and the force diagram would have looked like the one shown in Figure 13.6.

Observing that the x components add to zero (hence, no need to determine them) and the y components equal one another, the problem becomes nothing more than determining the y component of one of the forces and then doubling the quantity (there are two forces acting). Trickery is still required to determine the sine of the angle θ , but on the whole, the problem is not that complicated. Its solution is:

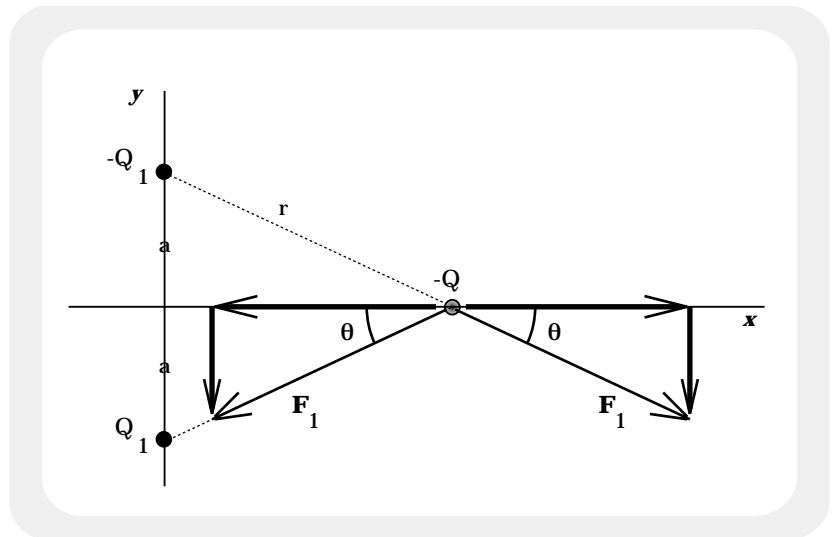


FIGURE 13.6

$$\mathbf{F}_Q = -2 \left[\frac{1}{4\pi\epsilon_0} \frac{QQ_1 a}{(a^2 + c^2)^{3/2}} \right] \mathbf{j}.$$

g.) Bottom line: As usual, memorizing this solution is next to useless. For the purpose of test-taking, you need to be able to approach a problem similar to this (possibly with a different orientation or an extra force) and:

i.) Determine the individual Coulomb force magnitudes due to all the charges acting on the charge-in-question;

ii.) Determine appropriate sine and cosine functions using known parameters within the system (i.e., a's and b's, etc.), and;

iii.) Put it all together in a vector sum.

5.) Example #4: Two weights of mass $m = .25$ kg each are attached to separate strings of length $L = .4$ meters and hung from a common point on the ceiling. When a charge q is placed on each mass, the masses repulse and swing out away from one another forming an angle $\theta = 22^\circ$ (see Figure 13.7a). What is the charge q ?

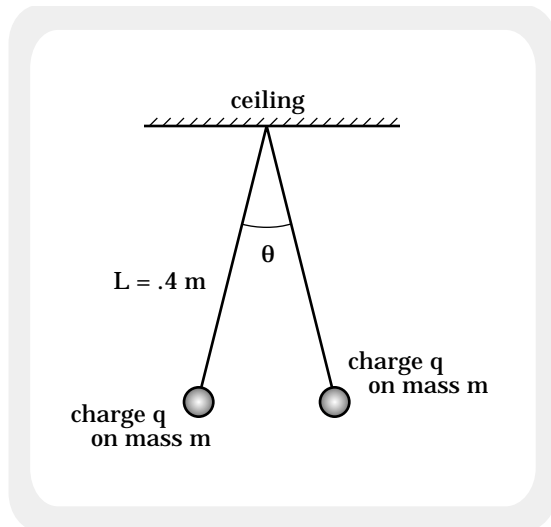


FIGURE 13.7a

a.) As we have static forces acting on two individual masses, we might find it useful to consider N.S.L. on one of the masses. An f.b.d. for the forces acting on the left mass is shown in Figure 13.7b below, where the repulsive force F_e is really a Coulomb force being generated by the right charge.

b.) Following through on Newton's Second Law:

$$\underline{\Sigma F_y:}$$

$$T \cos (\theta/2) - mg = ma_y = 0$$

$$\Rightarrow T = (mg)/(\cos \theta/2).$$

$$\underline{\Sigma F_x:}$$

$$T \sin (\theta/2) - F_e = ma_x = 0$$

$$\Rightarrow F_e = T \sin (\theta/2).$$

c.) Using F_e and eliminating T , we get:

$$\frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} = \frac{mg}{\cos\left(\frac{\theta}{2}\right)} \sin\left(\frac{\theta}{2}\right)$$

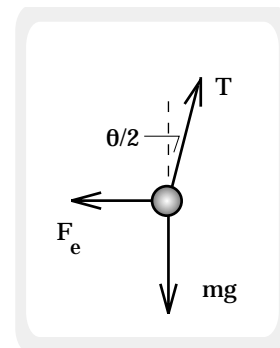


FIGURE 13.7b

$$\Rightarrow q = [mgr^2 \tan(\theta/2)/(1/4\pi\epsilon_0)]^{1/2}.$$

d.) Plugging in the numbers with $r = 2L\sin(\theta/2) = .153$ meters:

$$\begin{aligned} q &= \left[\frac{mgr^2 \tan\left(\frac{\theta}{2}\right)}{\frac{1}{4\pi\epsilon_0}} \right]^{1/2} \\ &= \left[\frac{(.25\text{kg})(9.8\text{m/s}^2)(.153\text{m})^2 \tan\left(\frac{22^\circ}{2}\right)}{9 \times 10^9 \text{ nt} \cdot \text{m}^2 / \text{C}^2} \right]^{1/2} \\ &= 1.11 \times 10^{-6} \text{ coulombs.} \end{aligned}$$

B.) Electric Fields:

1.) Electrical force is an important concept in electrical systems, but a more widely used concept is that of the **ELECTRIC FIELD**. An electric field is a modified force field. As a vector, it has a direction (this is defined as the direction in which a positive test charge would accelerate if put in the field at the point) and magnitude (the magnitude denotes the force per unit charge potentially available at the point) at every point in the field.

2.) The idea of a vector defining the "force per unit charge available at a point" probably seems odd, but there is a solid rationale for its existence. To understand why, consider the following analogy from the world of gravitational forces:

a.) According to Newton, massive objects produce "gravitational force fields" that affect other massive objects. And although the effects are noticeable only when at least one of the bodies is very large--an apple falling from a tree due to its gravitational interaction with the earth or the moon being centripetally accelerated into a nearly circular path due to its gravitational interaction with the earth--the phenomenon is characteristic of all massive objects.

b.) Newton derived an expression to determine how large this gravitational force was in a given case. When mass m_1 feels a gravitational force due to the presence of mass m_2 , the magnitude of the force is:

$$F_{\text{grav on mass 1}} = Gm_1m_2/r^2.$$

In this expression, the G term is called the Universal Gravitational Constant and r is the distance between the centers of mass of the two objects.

c.) Although the idea of a gravitational force field is much discussed (essentially due to all the work Newton did on the subject), the concept has a singular philosophic drawback. Force is a mass dependent quantity. If you know the gravitational force on a 65 kilogram object located 35,000 meters above the earth's surface, THAT IS ALL YOU KNOW! Inherent within that information is nothing about any other mass--the quantity is specific to a 65 kilogram mass located 35,000 meters above the earth, period.

d.) If we ignore the historical evolution of the topic, a more useful gravity-related quantity is the acceleration of a mass due to gravity. That quantity is universally appropriate to all masses; it is a mass independent quantity.

e.) Not clear? We know from Newton's Laws that if gravity is the only force acting on an arbitrary test mass m, the mass's gravitational acceleration is equal to:

$$a_g = F_g/m.$$

f.) There are two things to notice about this quantity. First, it is easy to experimentally test an acceleration field at a particular location to determine the gravitational acceleration at that point. All we have to do is take any mass m, use a Newton's Scale to measure the gravitational force F_g on the mass at the location of interest, then divide that force value by the size of the test mass (i.e., F_g/m). The resulting force per unit mass will be the magnitude of the gravitational acceleration at that point.

g.) Of considerably more importance is the inherent mass-independence wrapped up in the acceleration equation. It makes no difference how large the test mass m is. The acceleration a_g will always be the same at a given point. Acceleration really does measure the force per unit mass available at a given point.

h.) The point here is that because acceleration is a mass-independent quantity, the existence of a gravitational acceleration field is not dependent upon the presence of a mass to experience the field.

Acceleration fields exist whether masses actually are accelerating in the field or not. Gravitational force fields, on the other hand, do not have any meaning if there is not a mass to experience the force field within the system. Newton's gravitational force equation is specifically dependent upon the existence of such a mass.

i.) Bottom line: Gravitational acceleration fields are more primary in nature than are gravitational force fields.

3.) Just as a gravitational acceleration field is a vector field that tells you how much force per unit mass is available at a particular point in a gravitational force field, an ELECTRIC FIELD is a vector field that tells you how much force per unit charge is available at a particular point in an electric force field.

4.) Put another way, if we think of the earth's mass as generating a gravitational disturbance in the space around it--a disturbance that causes other massive objects to accelerate when placed in its field--we can think of a charged object as generating an electrical disturbance in the space around it--a disturbance that causes other charged objects to accelerate when placed in its field. That disturbance is mathematically embodied in the electric field vector.

5.) The mathematics in a nutshell:

a.) The magnitude of an electric field evaluated at a particular point is defined as:

$$E = F / q.$$

b.) The direction of an electric field at a particular point is defined as the direction a positive charge would accelerate if placed in the field at that point.

C.) The Electric Field Generated by a Point-Charge:

1.) How to determine an electric field vector EXPERIMENTALLY:

a.) Consider a specific example. Assume we have a point-charge Q sitting by itself in space. Being charged, it will affect other charges brought near it.

b.) To determine the magnitude of the electric field produced by Q at an arbitrary Point P :

- i.) Begin by placing a small, POSITIVE test charge (test charges are always positive!) at Point P;
 - ii.) Measure the force F_q the field exerts on the test charge;
 - iii.) Divide F_q by the size of the test charge q .
 - iv.) If done correctly, that calculation will yield the magnitude of the force per unit charge available at the Point P. That is, the magnitude of Q's electric field evaluated at Point P.
- c.) To determine the direction:
- i.) Allow a positive test charge to accelerate freely (there cannot be any other forces acting on the charge at the time) in the electric field.
 - ii.) The direction of the acceleration of the test charge is the direction of the electric field at Point P.

2.) How to determine an electric field magnitude theoretically:

- a.) In the situation outlined in Part 1 above, assume the test charge is q and the distance between the test charge and the field-producing charge Q is r .
- b.) Coulomb's Law allows us to determine the magnitude of the electrical force on the test charge q while near Q . That is:

$$F_q = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2}.$$

- c.) Dividing F_q by the size of the test charge, we get:

$$\begin{aligned} E_Q &= \frac{F_q}{q} \\ &= \frac{\frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2}}{q} \\ &= \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}. \end{aligned}$$

d.) This is the magnitude of the electric field as evaluated a distance r units from the field-producing point charge Q .

Note 1: Notice that this expression is a function only of the field-producing charge Q and the distance r between the charge and the point of interest. Not only is the size of Q 's electric field not affected by the size of the test charge used to probe it, Q 's electric field exists whether there is a secondary charge experiencing the field or not.

Note 2: There are Calculus-driven techniques for deriving theoretical expressions for electric fields generated by exotic charge configurations (all we have worked with are point charges). We will deal with them shortly.

3.) Example #1: A -4×10^{-12} coulomb charge Q is located at the origin. What is the electric field (as a vector) produced by the charge at the coordinate $x = -.02$ meters?

a.) The magnitude of the field will be:

$$\begin{aligned} E_Q &= \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \\ &= (9 \times 10^9 \text{ nt} \cdot \text{m}^2 / \text{C}^2) \frac{(4 \times 10^{-12} \text{ C})}{(.02 \text{ m})^2} \\ &= 90 \text{ nts} / \text{C}. \end{aligned}$$

Note: Notice that the sign of Q in the equation is positive even though the charge is a negative one. Just as was the case with Coulomb's Law, this equation allows you to determine the magnitude of the electric field only. As such, only charge magnitudes are used in it.

b.) The direction of the field: An electric field's direction at a point in space is defined as the direction a positive charge would be accelerated if put in the field at the point in question. In this case, a positive test charge placed at $x = -.02$ meters (i.e., 2 cm to the left of the origin) would feel an electric force in the $+i$ direction. By definition, that is the direction of the electric field.

c.) Putting it all together, the electric field produced by the -4×10^{-12} coulomb charge at $x = -.02$ meters is $E_Q = (90 \text{ nt/C})i$.

4.) Example #2: Assume an electric field $E = (+90 \text{ nt/C})i$ permeates the area surrounding the origin. When a charge $q_1 = -3 \times 10^{-17}$ coulombs is placed at $x_1 = -.02$ meters (i.e., to the left of the origin), that charge will feel a force. As a vector, how large will that force be?

a.) We know that the relationship between the force F on a charge q in an electric field E is $F = qE$.

b.) If the charge was positive ($+q$), the process would be simple because the direction of F on q would, by definition, be the same as the direction of E . As such, we could use $E = F/q$ to write:

$$\begin{aligned} F &= q E \\ &= (+3 \times 10^{-17} \text{ C}) [(90 \text{ nt / C}) i] \\ &= (2.7 \times 10^{-15} \text{ nts}) i. \end{aligned}$$

c.) What if the charge is negative? Technically, we should multiply the magnitude of the electric field by the magnitude of the charge to get the magnitude of the force, or $qE = (3 \times 10^{-17} \text{ C})(90 \text{ nt/C}) = 2.7 \times 10^{-15} \text{ nts}$.

To get the force direction, we know that the electric field's direction ($+i$ in this case) is defined as the force direction on a positive charge, so we know that the opposite direction will be the force direction on a negative charge. In short, $F = (2.7 \times 10^{-15} \text{ nts})(-i)$. Alternately, this can be written as $F = (-2.7 \times 10^{-15} \text{ nts})(i)$.

Note: If we had used $F = qE$ with the charge's sign, we would have gotten the same answer. That is, $F = (-3 \times 10^{-17} \text{ C})(90 \text{ nt/C}) i = (-2.7 \times 10^{-15} \text{ nts})i$. THIS ISN'T TECHNICALLY KOSHER, as a charge's sign should never be used to determine directional quantities, but due to the way electric field directions are defined (i.e., the direction a positive charge will be forced . . . etc.), we get the right answer. You can use this approach, but only with the understanding that you are fudging things in doing so.

5.) Electric fields superimpose vectorially. If a number of charges are placed close to one another, a net electric field will be generated at every point in space in the vicinity of the group. That net electric field will be the vector sum of each charge's electric field as it exists at the point of interest.

To see this, consider a fixed charge $-Q_1$ located a distance a units up the y -axis and a second fixed charge Q_2 located a distance b units down the y -axis. What is the electric field (as a vector) a distance c units down the $+x$ -axis? (See Figure 13.8a on the next page).

a.) Remember, electric fields are not scalars--they have to be treated like VECTORS. In fact, this problem is very similar to the electric force problem done in Part A-4. The fundamental difference is that the vectors involved here are electric field vectors.

b.) We need to determine:

- i.) The direction of each electric field acting at $(c, 0)$.
- ii.) The magnitude of each electric field acting at $(c, 0)$.
- iii.) The x and y components of each field acting at $(c, 0)$.
- iv.) The vector sum of those components.

c.) Electric field directions at Point $(c, 0)$:

i.) E. fld. due to $-Q_1$:
As a positive test charge would be accelerated toward $-Q_1$, the direction of the electric field at c due to that charge will be toward $-Q_1$ along the line between $-Q_1$ and Point $(c, 0)$ --(see Figure 13.8a).

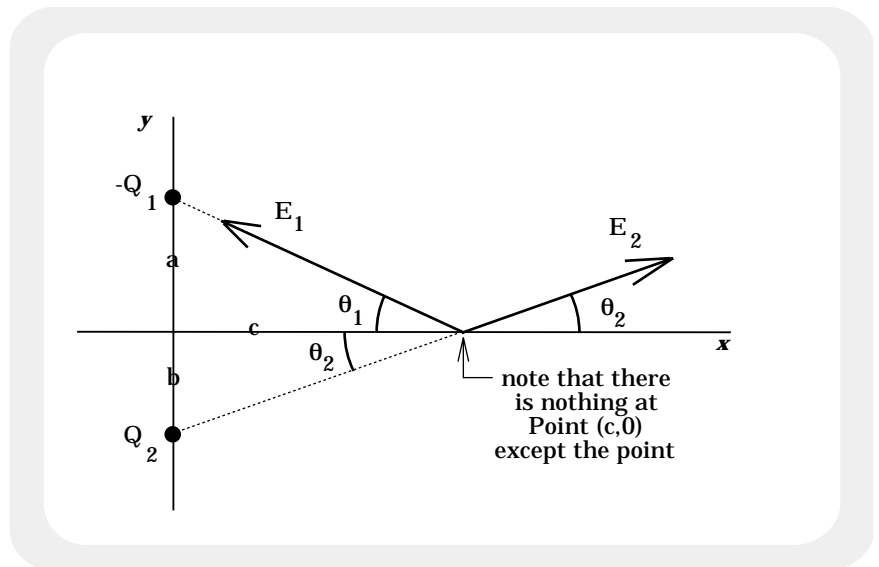


FIGURE 13.8a

ii.) E. fld. due to Q_2 : The direction of the electric field at Point $(c, 0)$ due to the presence of Q_2 is away from Q_2 along a line between the point and Q_2 (see Figure 13.8a).

Note: These electric field lines are exactly the opposite of the force field lines shown in Figure 13.5a even though the charge distributions are the same in both cases. The difference: Figure 13.5a showed the electric forces on a negative charge located at Point $(c, 0)$; Figure 13.8a shows the electric

field vectors at Point (c,0). An electric field direction is defined as the direction in which a POSITIVE charge would accelerate if set free in the field at the point-of-interest. Because we are dealing with a negative charge in the first case and an implied positive charge in the second case, the two figures are mirror images of one another.

d.) The magnitude of each electric field is determined using our equation for the E. fld. magnitude of a point charge:

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r_1^2} \quad \text{and} \quad E_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_2}{r_2^2},$$

where $r_1 = (a^2 + c^2)^{1/2}$ and $r_2 = (b^2 + c^2)^{1/2}$.

Note: The charge values used in these equations do not include their signs (i.e., all are treated as being positive). That is, the equations allow us to calculate magnitudes only. The directional nature of each electric field must be determined by eyeballing the situation in relationship to the coordinate axes provided within the system.

e.) To put the electric fields into component form, we need to determine the sine and cosine of both θ_1 and θ_2 . As we haven't been given those angles, we need to use the same kind of trickery as before to determine those sine and cosine quantities in terms of variables we know (i.e., terms like a, b, or c). Using the right triangles shown in Figure 13.8a, we find:

i.) For $\sin \theta_1$:

$$\begin{aligned} \sin \theta_1 &= (\text{opp. side})/(\text{hyp.}) \\ &= a/(a^2 + c^2)^{1/2}. \end{aligned}$$

ii.) For $\cos \theta_1$:

$$\begin{aligned} \cos \theta_1 &= (\text{adj. side})/(\text{hyp.}) \\ &= c/(a^2 + c^2)^{1/2}. \end{aligned}$$

iii.) Similarly:

$$\begin{aligned} \sin \theta_2 &= b/(b^2 + c^2)^{1/2} \quad \text{and} \\ \cos \theta_2 &= c/(b^2 + c^2)^{1/2}. \end{aligned}$$

f.) We are now in a position to write vectorially the sum of the electric fields acting at Point $(c,0)$. This becomes:

$$(-E_1 \cos \theta_1 + E_2 \cos \theta_2) \mathbf{i} + (E_1 \sin \theta_1 + E_2 \sin \theta_2) \mathbf{j}.$$

i.) Expanding the magnitude of $E_1 \cos \theta_1$ (see Figure 13.8b below), we get:

$$\begin{aligned} E_{1,x} &= E_1 \cos \theta_1 \\ &= \frac{1}{4\pi\epsilon_0} \frac{Q}{(a^2 + c^2)^{3/2}} \left[\frac{c}{(a^2 + c^2)^{1/2}} \right] \\ &= \frac{1}{4\pi\epsilon_0} \frac{Qc}{(a^2 + c^2)^{3/2}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{Qc}{r_1^3}. \end{aligned}$$

Note 1: The defined value of r_1 was $r_1 = (a^2 + c^2)^{1/2}$. It has been used above because things are about to get messy-- brevity here will help.

Note 2: Reiteration: Although it has already been mentioned above, do not become confused between our use of $E = kQ/r^2$ to determine the magnitude of one of the many charges contributing to the net

electric field and our previous use of the equation $F = qE$. The latter assumes we already know the net electric field as a vector at the point-of-interest. In such a case, determining the force on a charge must be done by including the charge's sign when using $F = qE$. As was stated above, the reasoning behind that move is simple: the electric field direction and the force field direction are the same if the affected charge is positive; they are opposite if the charge

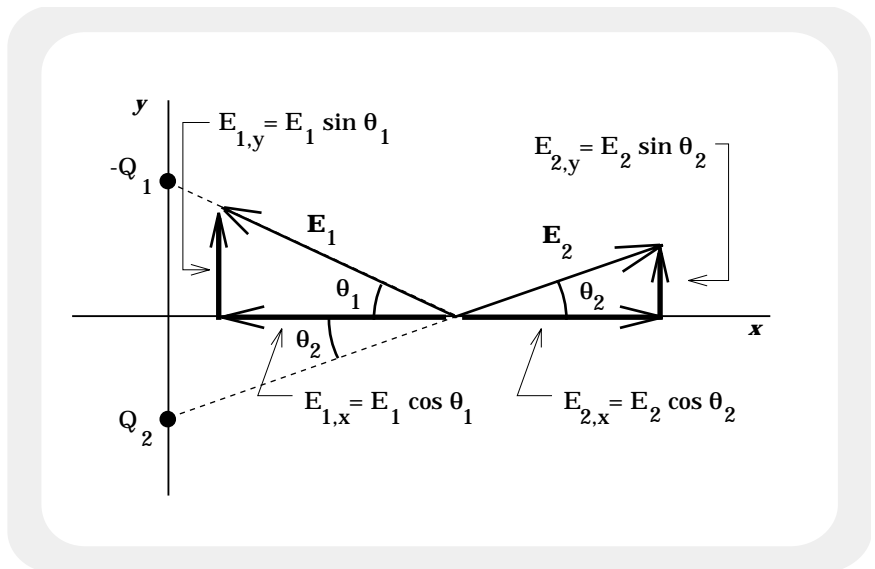


FIGURE 13.8b

feeling the force is negative. Including the charge's sign in qE ensures the correct direction for F in all cases.

In the situation we are currently working with, WE DO NOT KNOW THE NET ELECTRIC FIELD VECTOR at the point-of-interest (that is what we are looking for). As such, we must determine all field DIRECTIONS by eyeballing and all field MAGNITUDES using kQ/r^2 . Being magnitudes, all Q values used in kQ/r^2 must be positive.

ii.) In expanded form (including "eyeballed" signs), the net electric field vector becomes:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \left[\left(-\frac{Q_1 c}{(a^2 + c^2)^{3/2}} + \frac{Q_2 c}{(b^2 + c^2)^{3/2}} \right) \mathbf{i} + \left(\frac{Q_1 a}{(a^2 + c^2)^{3/2}} + \frac{Q_2 b}{(b^2 + c^2)^{3/2}} \right) \mathbf{j} \right].$$

g.) Just as was the case with the force problem examined in Part A, this exercise would have been considerably easier if there had been more symmetry. If the charge magnitudes had been equal (Q) and the distances a and b equal (r), then both electric fields E_1 and E_2 would have had the same magnitude and angle and the electric field diagram would have looked like the one shown in Figure 13.9.

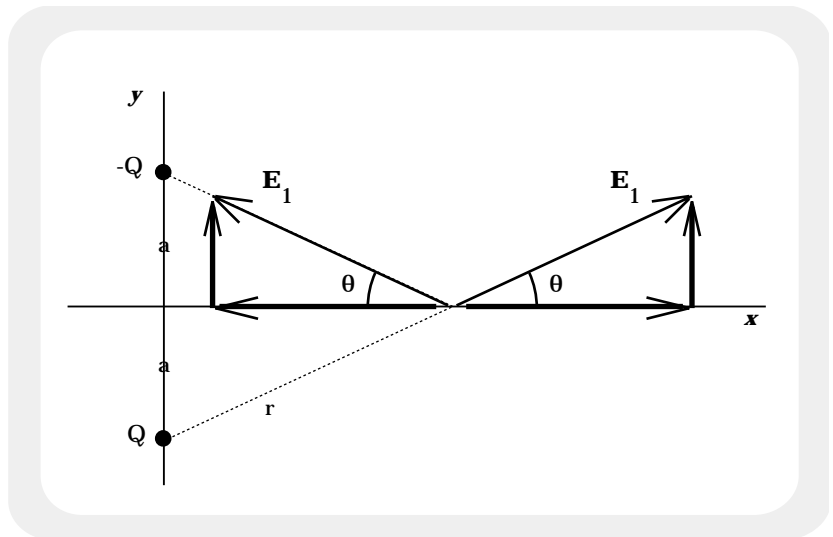


FIGURE 13.9

In this case, as before, the x components add to zero, the y components equal one another, and the net electric field becomes:

$$\mathbf{E} = 2 [Qa/(4\pi\epsilon_0 r^3)] \mathbf{j} .$$

D.) Electric Field Lines:

1.) So far, we have dealt with the mathematics of electric fields generated by point-charges evaluated at specific positions in space. As useful as this is, there is a more general way of representing electric fields that does not depend so heavily upon mathematics.

2.) Consider a point-charge Q . The arrows in Figure 13.10 show the directions a positive test charge would accelerate if put at various points in Q 's field. In other words, we have schematically depicted the direction of Q 's electric field at selected points. These lines are called electric field lines.

3.) Assuming the electric field lines are constructed symmetrically about the field-producing charge, they tell us two things:

a.) The arrows give us a sense of the direction of the electric field in a particular region (i.e., the field is upward swooping to the left, or whatever), and;

b.) The relative distance between the lines gives us a sense of the intensity of the field in a particular region. That is, just as a topographic map sketches elevation lines that are close together where the geography is very steep and far apart where the geography is flat, electric field lines are close together where the electric field is very large and far apart where the electric field is small.

i.) This can easily be seen by looking at our sketch in Figure 13.10. Close to Q the electric field is large and the field lines are very close together; further out the electric field is small and the field lines are spread out.

c.) In a nutshell: the information provided by electric field lines is qualitative, not quantitative. If you know how to interpret them, they give you a relative reading as to the intensity and direction of an electric field in a particular region.

4.) How to determine electric field lines if all one has is a sketch of the charge distribution:

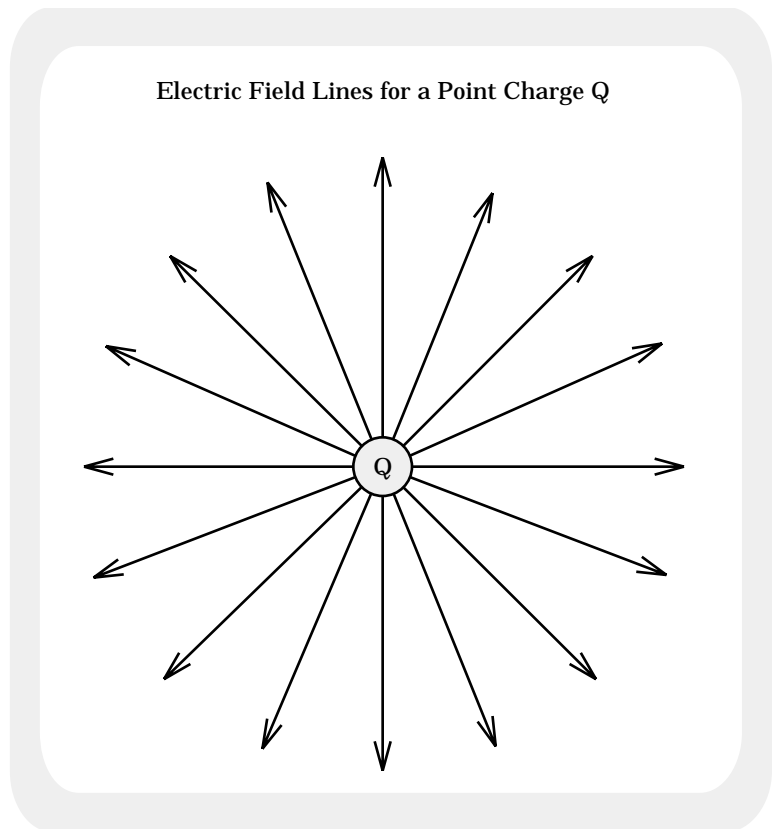


FIGURE 13.10

a.) Consider the two equal, positive charges:

b.) As has been done in Figure 13.11a, start at one of the positive charges and place either eight or sixteen symmetric dots around the charge (these dots will be used as starting places--the number of dots isn't important but symmetry is if the resulting field lines are to have any significance).

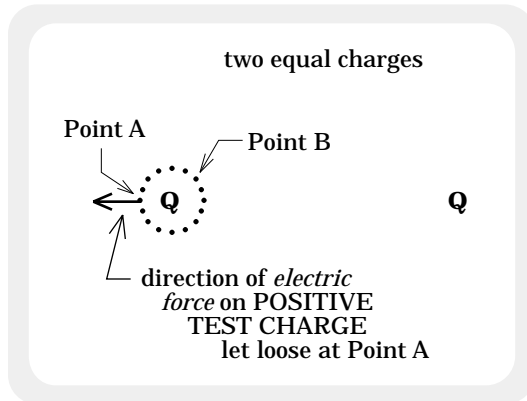


FIGURE 13.11a

c.) Begin at a convenient dot (Point A on the sketch will do) and ask the question: "If placed at this point, in what direction will a positive test charge accelerate as a consequence of its interaction with the fixed, field-producing charges (in this case, the two Q's)?"

d.) For Point A, both field-producing charges will push a positive test charge to the left, so the electric field vector at Point A will be in the $-i$ direction. Make a mental note of that.

Next, move a short distance along the line-of-force as experienced at Point A and, upon coming to a likely spot, mentally place a positive test charge at that position. Ask, "In what direction will a positive test charge accelerate if put at this position?" Make a mental note of your conclusion (in this case, due to the symmetry of the situation, the field vector will again be in the $-i$ direction).

This is all summarized in Figure 13.11a.

e.) Take a second point, say Point B in Figure 13.11a, and repeat the query quoted in Part d. In this case, the left-hand field-producing charge will push the test charge to the right and upward while the right-hand field-producing charge will push the test charge to the left and upward. The left-hand charge is

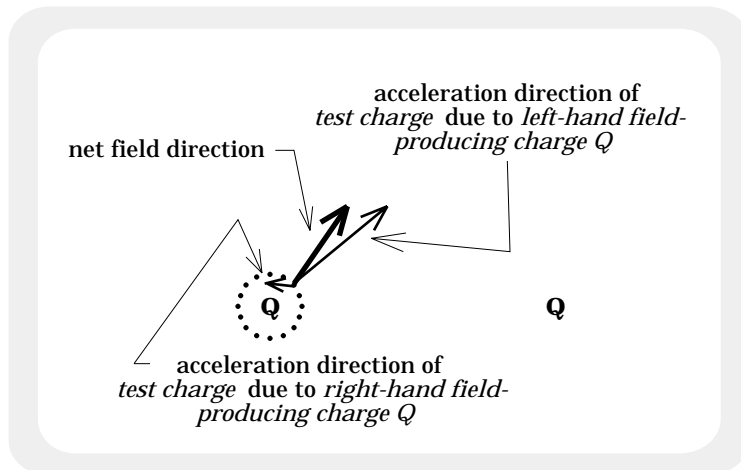


FIGURE 13.11b

closer so its push will be greater. The net push will be in the direction approximated in Figure 13.11b on the previous page.

f.) Continue this process for all eight dots, using any symmetry that might make your task easier (if you know the field lines at the top five dots of the left-hand charge, you not only know the bottom dots of that charge but also all the lines around the right-hand charge). The final configuration is shown in Figure 13.11c.

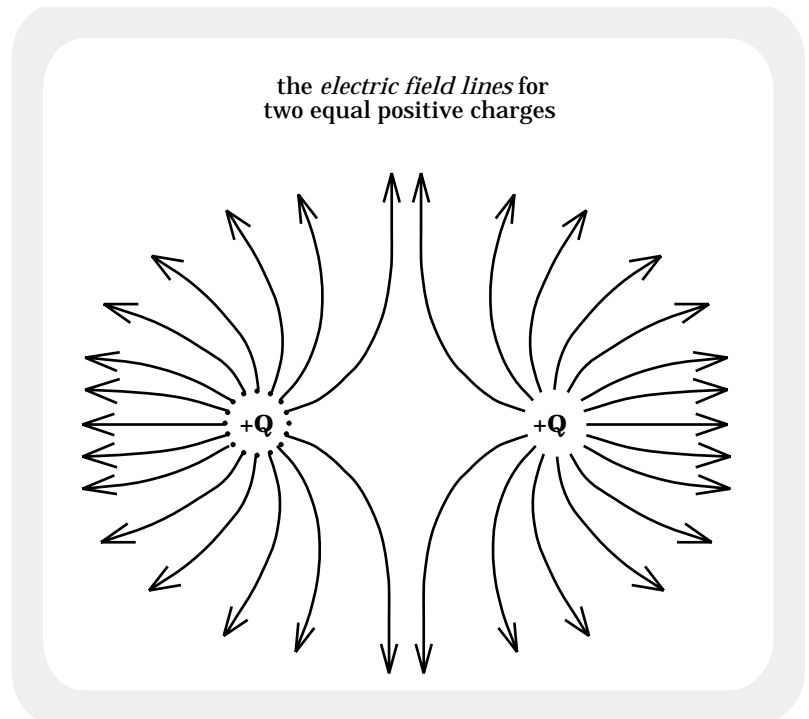


FIGURE 13.11c

E.) Electric Fields and Extended Charge Configurations:

1.) To this point, all we have talked about have been point charges. How does one deal with extended objects that are charged? That is what we are about to examine.

2.) Consider a circular hoop of radius R with total charge Q distributed uniformly on its surface. Derive an expression for the electric field intensity (i.e., the electric field magnitude) a distance b units down the x -axis (see Figure 13.12).

a.) Begin by defining a differential charge dq at some arbitrary position on the hoop. That differential charge will produce a differential electric field dE at $x = b$.

b.) Figure 13.13a shows the differential charge dq positioned at

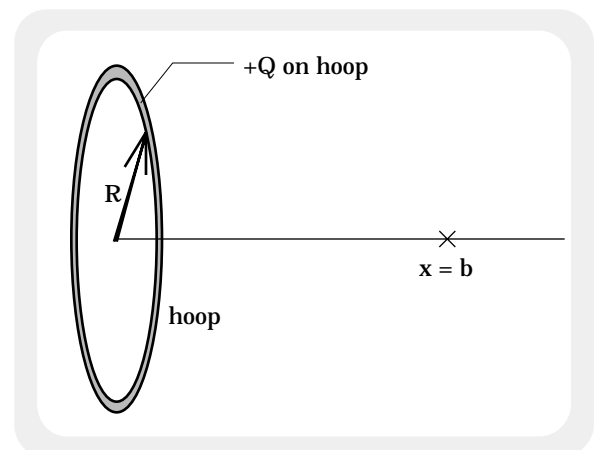


FIGURE 13.12

the top of the hoop and the differential electric field at $x = b$ pictured with components. Additionally, the distance between dq and $x = b$ is defined as r and is presented in terms of the known variables R and b , and the sine and cosine of the angle θ is defined.

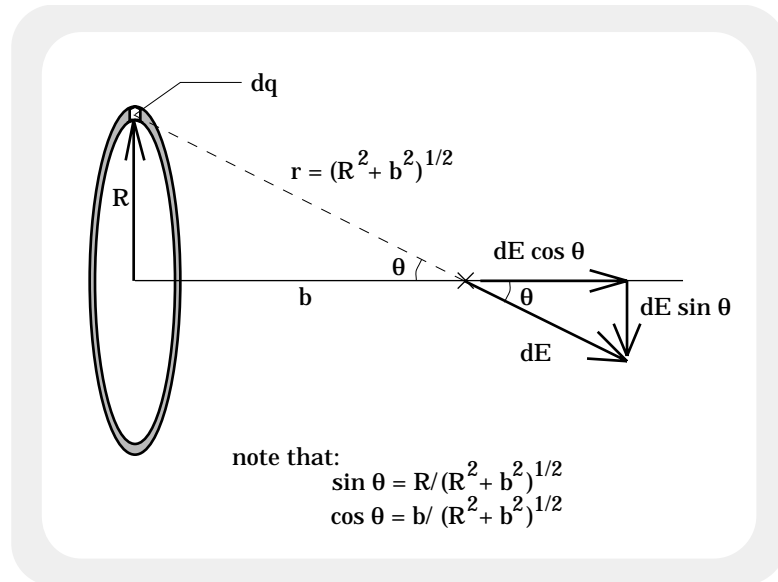


FIGURE 13.13a

c.) We could determine the x and y components of dE , then integrate each to determine the i and j parts of the net electric field at $x = b$. Or, we could be clever.

d.) The being clever approach: On the underside of the hoop, there is another field-producing charge dq whose electric field at $x = b$ exactly mirrors the field produced by our original dq (see this in the side-view of the hoop presented in Figure 13.13b). Notice that when the two fields are added together, the y components add to zero. Put in a little different context, if we derive an expression for $dE \sin \theta$, then integrate it, we will end up with zero.

Knowing this, we can forget the y components and focus solely on the x components of the field.

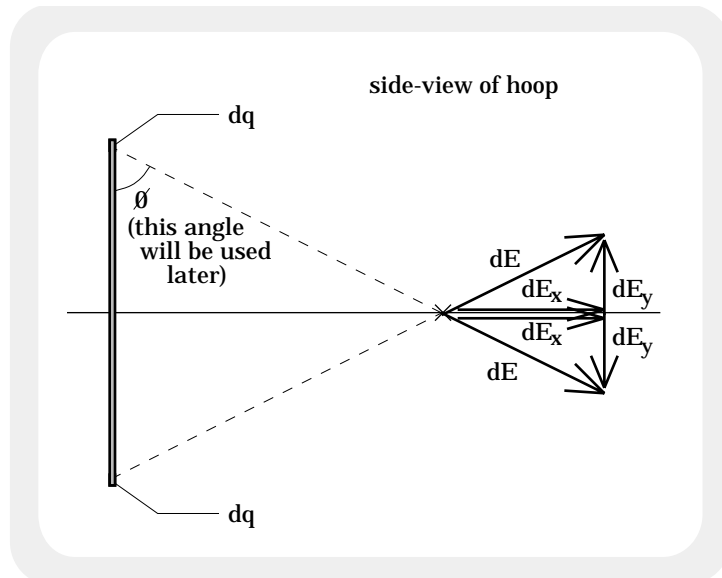


FIGURE 13.13b

e.) The differential electric field in the x direction is:

$$\begin{aligned} dE_x &= \left[\frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \right] \cos\theta \\ &= \left[\frac{1}{4\pi\epsilon_0} \frac{dq}{(R^2 + b^2)^{1/2}} \right] \left(\frac{b}{(R^2 + b^2)^{1/2}} \right), \\ &= \left[\frac{b}{4\pi\epsilon_0} \frac{dq}{(R^2 + b^2)^{3/2}} \right], \end{aligned}$$

where dq , r , and θ are defined above. The only parameter in this expression that varies is dq . Noting that $\int dq$ equals the total charge Q on the hoop (all we are doing is adding all the differential charges on the hoop, the sum of which must be the total charge on the hoop), we can write:

$$\begin{aligned} \mathbf{E} &= \left[\int dE_x \right] \mathbf{i} \\ &= \left[\frac{b}{4\pi\epsilon_0 (R^2 + b^2)^{3/2}} \int dq \right] \mathbf{i} \\ &= \left[\frac{bQ}{4\pi\epsilon_0 (R^2 + b^2)^{3/2}} \right] \mathbf{i}. \end{aligned}$$

f.) Note that if we allow b to become very large, the electric field should look like that of a point charge. Why? Because a hoop at great distance looks like a point. Setting $b \gg R$, we find that our expression reduces to $\frac{1}{4\pi\epsilon_0} \frac{Q}{b^2}$. This is, in fact, the expression for the electric field generated by a point charge.

3.) It is not difficult to extend the hoop problem to that of a flat disk. To make the problem more interesting, though, we will assume that the charge on our disk is not uniformly distributed. In fact, let's assume it has a surface charge density (i.e., a charge per unit area) of $\sigma = (Q/R)r$, where r is the distance from the center of the disk to the point of interest, Q is the total charge on the disk, and R is the disk's radius. With those assumptions, derive an expression for the electric field a distance b units down the x-axis.

a.) Consider a hoop of radius r and thickness dr on the disk's face (see Figure 13.14). We already have an expression for the magnitude and direction of the electric field along the x -axis produced by a charged hoop. Acknowledging that the field is in the x direction and calling the differential charge on the hoop dq , we can fit that expression to this problem and write (without vector notation):

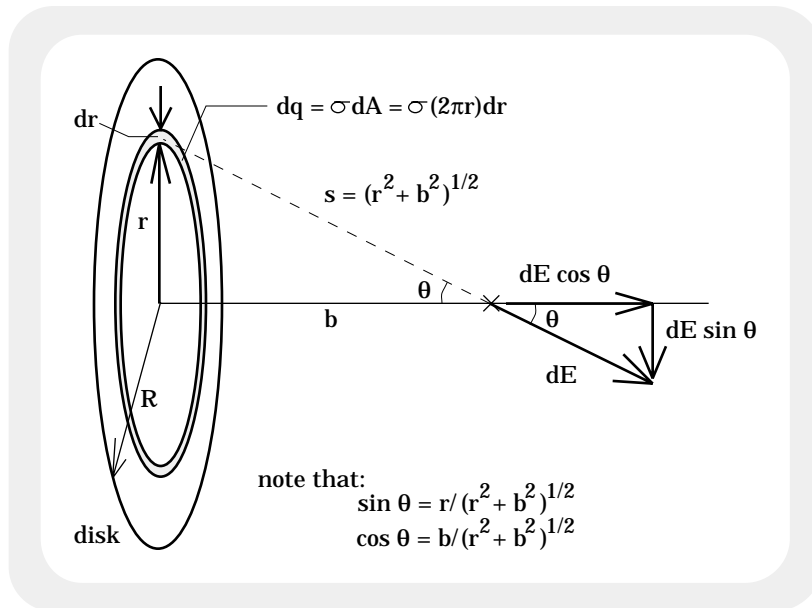


FIGURE 13.14

$$dE_x = \left[\frac{(b)dq}{4\pi\epsilon_0(r^2 + b^2)^{3/2}} \right].$$

b.) The only parameters that vary in this expression are dq and r . To eliminate one of those parameters, we need to express dq in terms of dr . We can do this using the surface charge density function $\sigma = (Q/R)r$. With that, we can write:

$$dq = \sigma dA,$$

where dA is the differential surface area of the hoop. Mathematically, this will be the circumference of the hoop times its thickness, or:

$$dA = (2\pi r)dr.$$

Using dA and substituting in $\sigma = (Q/R)r$, we get:

$$\begin{aligned} dq &= \sigma dA \\ &= [(Q/R)r][(2\pi r)dr] \\ &= (2\pi Q/R)r^2 dr. \end{aligned}$$

c.) Substituting dq into our electric field expression, we get:

$$\begin{aligned} dE_x &= \frac{(b)}{4\pi\epsilon_0 (r^2 + b^2)^{3/2}} dq \\ &= \frac{(b)}{4\pi\epsilon_0 (r^2 + b^2)^{3/2}} \left[\frac{2\pi Q}{R} r^2 dr \right]. \end{aligned}$$

d.) Remembering that this field will be in the $+i$ direction and noting that the limits of integration will be $r = 0$ and $r = R$, we can do some canceling, re-write the expression, and solve:

$$\begin{aligned} E &= \int dE_x \\ &= \frac{bQ}{2\epsilon_0 R} \int_{r=0}^R \frac{r^2}{(r^2 + b^2)^{3/2}} dr \\ &= \frac{bQ}{2\epsilon_0 R} \left[-\frac{r}{(r^2 + b^2)^{1/2}} + \ln[r + (r^2 + b^2)^{1/2}] \right]_{r=0}^R \\ &= \frac{bQ}{2\epsilon_0 R} \left[\left(-\frac{R}{(R^2 + b^2)^{1/2}} + \ln[R + (R^2 + b^2)^{1/2}] \right) - (\ln b) \right] \\ &= \frac{bQ}{2\epsilon_0 R} \left[-\frac{R}{(R^2 + b^2)^{1/2}} + \ln\left(\frac{R + (R^2 + b^2)^{1/2}}{b}\right) \right]. \end{aligned}$$

Note: It is unlikely you would have known this integral. As usual, you could have dug deep into your bag of Calculus tricks and figured some exotic way of doing the integral, or you could have used a book of integrals. One way or the other, 95% of a test problem like this is in setting up the problem. Evaluation of the integral is just gravy.

e.) If we hadn't already determined the expression for a hoop, we would have had to do the above problem from scratch. To see what that would have looked like, follow along:

i.) The charge dq in a hoop of radius r and thickness dr will be the surface charge density times the hoop's differential area, or:

$$\begin{aligned} dq &= \sigma \quad dA \\ &= [(Q/R)r][2\pi r dr] \end{aligned}$$

$$= \frac{2\pi Q}{R} r^2 dr.$$

ii.) The differential electric field dE in the x direction (in this case, the net field generated by the charges of the hoop) will be:

$$\begin{aligned} dE_x &= \left[\frac{1}{4\pi\epsilon_0} \frac{dq}{s^2} \right] \cos\theta \\ &= \left[\frac{1}{4\pi\epsilon_0} \frac{\sigma dA}{s^2} \right] \cos\theta \\ &= \left[\frac{1}{4\pi\epsilon_0} \frac{\left[\left(\frac{2\pi Q}{R} \right) r^2 dr \right]}{(r^2 + b^2)^{1/2})^2} \right] \left(\frac{b}{(r^2 + b^2)^{1/2}} \right) \\ &= \left[\frac{bQ}{2\epsilon_0 R} \right] \left(\frac{r^2}{(r^2 + b^2)^{3/2}} \right) dr. \end{aligned}$$

iii.) This is the same expression we integrated above.

4.) Are there steps to attacking problems like these? Yes!

a.) Everything springs from the electric field expression for a point charge, or $\frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$.

b.) In such cases, define a differential point charge dq , then determine the electric field (as a vector) it generates at the point of interest. Knowing the differential field for one, small, arbitrarily defined bit of charge, integrate it to determine the total field produced by all the bits of charge.

c.) If there is symmetry, exploit it to cut down on the number of integrals required for evaluation.

d.) If the charge is defined in terms of a surface charge density function (or a linear charge density function, or even a volume charge density function), use that variable to relate dq to the geometry of the structure.

5.) Another problem:
 Consider a rod of length $2a$ that has $+Q$'s worth of charge uniformly distributed over its upper half and $-Q$'s worth of charge uniformly distributed over its bottom half (see Figure 13.15). Derive an expression for the electric field intensity a distance b units down the x -axis.

a.) Begin by defining a differential charge dq at some ARBITRARY position a distance y units up the axis. That differential charge will produce a differential electric field dE at the point of interest.

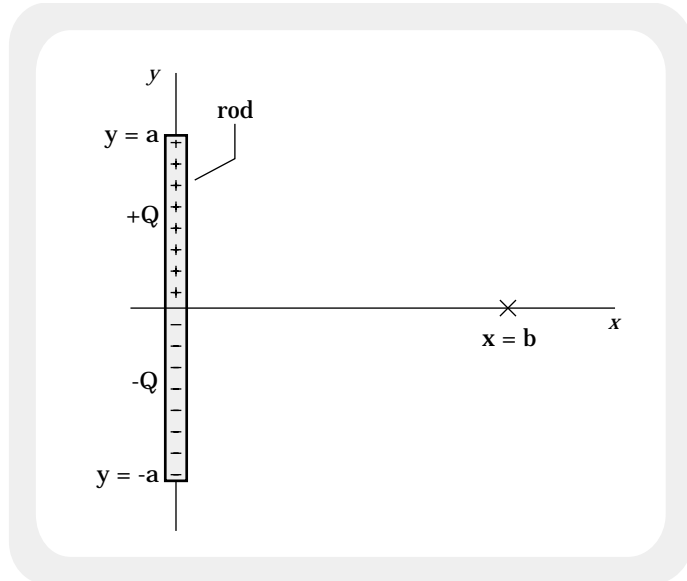


FIGURE 13.15

Note 1: We are trying to generate an expression for the differential electric field at $x = b$ due to any differential charge on the rod. We need it because we want to integrate over ALL dq 's. Never take the charge at an endpoint or any other special point to be your arbitrarily chosen, differential point charge. If you do, quantities like y that should be variables will be replaced by specific values (i.e., $y = a$) and the resulting expression will not be general.

Note 2: Notice that y will vary from point to point along the rod, which means the distance r between any given dq and $x = b$ will vary.

b.) Figure 13.16a shows a differential charge dq on a differential length of rod dy positioned at an arbitrarily chosen coordinate y .

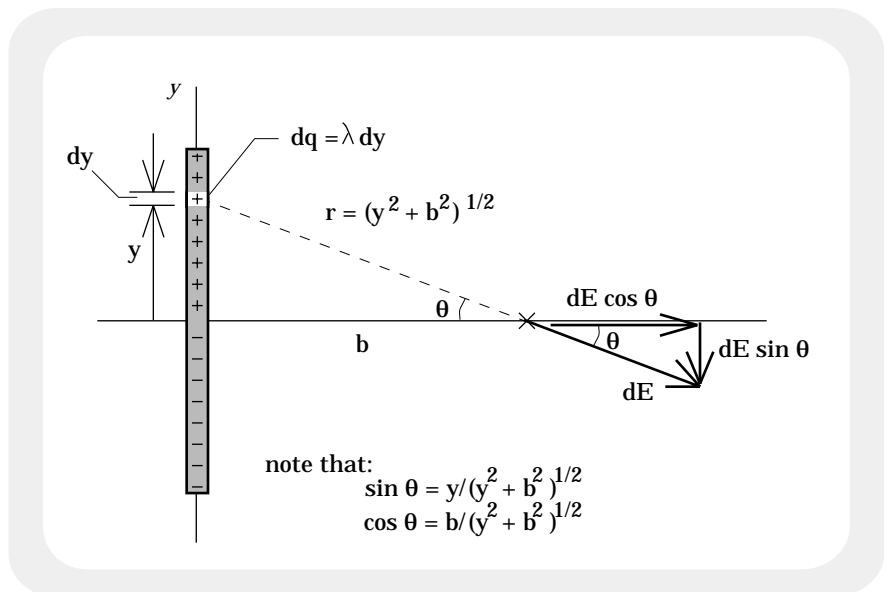


FIGURE 13.16a

The differential electric field dE at $x = b$ is pictured with components. Additionally, the distance r between dq and $x = b$ is defined and presented in terms of the known parameter b and the variable y , and the sine and cosine of the angle θ is defined.

c.) As was the case in the previous problem, it is to our advantage to identify and exploit any symmetry within the system. With that in mind, consider the differential field produced by the charge dq located at coordinate $-y$ (see Figure 13.16b).

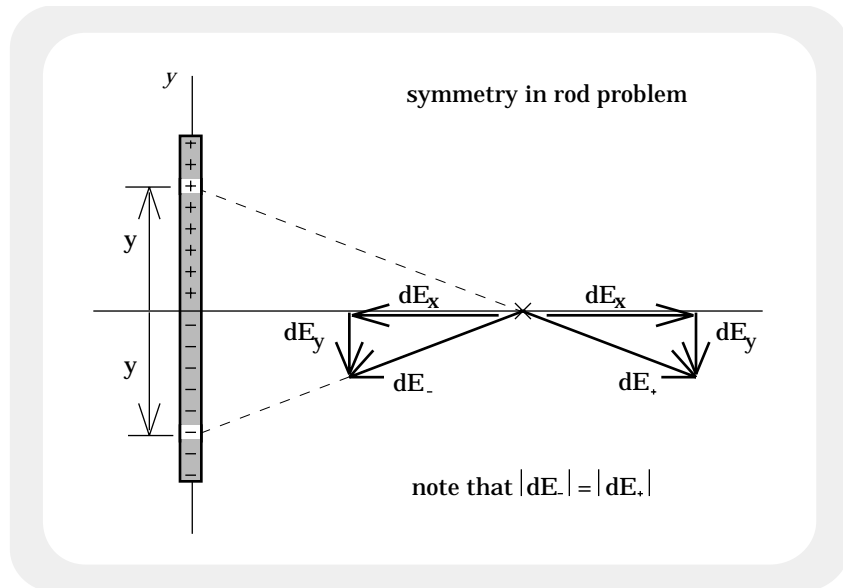


FIGURE 13.16b

Looking at the figure, it is evident that the x components will add to zero in this case, which means we can ignore that direction and focus all of our attention on the y components of the field.

d.) The y component of the electric field generated by dq at $x = b$ will be:

$$dE_y = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \sin \theta.$$

e.) Using $\sin \theta = y/(y^2 + b^2)^{1/2}$ and $r = (y^2 + b^2)^{1/2}$, this becomes:

$$\begin{aligned} dE_y &= \frac{1}{4\pi\epsilon_0} \frac{dq}{(y^2 + b^2)^{1/2})^2} \left[\frac{y}{(y^2 + b^2)^{1/2}} \right] \\ &= \frac{dq}{4\pi\epsilon_0} \frac{y}{(y^2 + b^2)^{3/2}}. \end{aligned}$$

f.) What varies in this problem is y . Unfortunately, the above expression is in terms of dq , not dy . That is, we need to express dq in terms of dy .

To do so, let's define a linear charge density function (i.e., the amount of charge per unit length on either the upper or lower half of the rod). We can do this in two ways:

i.) Macroscopically: Take the total charge Q on the upper half of the rod and divide by the length a of that section, or:

$$\lambda = Q/a.$$

ii.) Microscopically: Take the differential charge dq and divide it by the length of the section of rod upon which it sits, or:

$$\begin{aligned}\lambda &= dq/dy \\ \Rightarrow dq &= \lambda dy.\end{aligned}$$

iii.) Coupling the two expressions for λ , we can write:

$$dq = (Q/a)dy.$$

g.) Due to symmetry, doubling the net y component of the field produced by the positive charge on the upper half of the rod will yield the magnitude of the total electric field. Doing so:

$$\begin{aligned}E &= 2 \int_y dE \\ &= (2) \frac{1}{4\pi\epsilon_0} \int_{y=0}^a \frac{y}{(y^2 + b^2)^{3/2}} dq \\ &= \frac{1}{2\pi\epsilon_0} \int_{y=0}^a \frac{y}{(y^2 + b^2)^{3/2}} \left(\frac{Q}{a} dy \right) \\ &= \frac{Q}{2\pi\epsilon_0 a} \int_{y=0}^a \frac{y}{(y^2 + b^2)^{3/2}} dy \\ &= \frac{Q}{2\pi\epsilon_0 a} \left[\frac{-1}{(y^2 + b^2)^{1/2}} \right]_{y=0}^a \\ &= \frac{Q}{2\pi\epsilon_0 a} \left[\left(\frac{-1}{(a^2 + b^2)^{1/2}} \right) - \left(\frac{-1}{(b^2)^{1/2}} \right) \right] \\ &= \frac{Q}{2\pi\epsilon_0 a} \left[\frac{1}{b} - \left(\frac{1}{(a^2 + b^2)^{1/2}} \right) \right].\end{aligned}$$

F.) Shielding:

1.) Consider a flat, conducting plate that is attached to a generator designed to force negative charge (electrons) onto the plate (see Figure 13.17).

a.) When operating, the generator continues to force negative charge onto the plate until electrostatic repulsion finally stops the charge flow.

b.) Because the plate is flat, every charge on the plate will repulse every other charge on the plate.

c.) When fully charged, the plate will have a relatively uniform charge density σ_1 on its surface.

2.) Consider that same flat conducting plate mentioned above, but assume it is uniformly curved (see Figure 13.18). Again, the electron generator is attached.

a.) When operating, the generator again forces negative charge onto the plate until electrostatic repulsion finally stops the charge flow.

b.) Because the plate is curved, every charge on the plate will not repulse every other charge on the plate. Why? Because there is material separating the charges. This effect is called shielding.

c.) When fully charged, the plate will have a relatively uniform charge density σ_2 on its surface. But because the electrons on the right side of the plate do not affect the electrons on the left side, more electrons can be forced onto the curved plate than onto the flat plate. As such, the curved plate's surface charge density σ_2 turns out to be greater than σ_1 .

3.) Things get interesting when dealing with pointed conductors (see Figure 13.19) . . . (note that the following is also true of ridged conductors). When

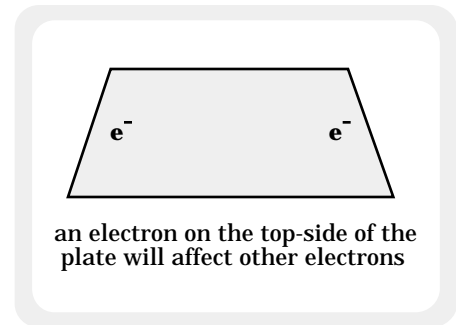


FIGURE 13.17

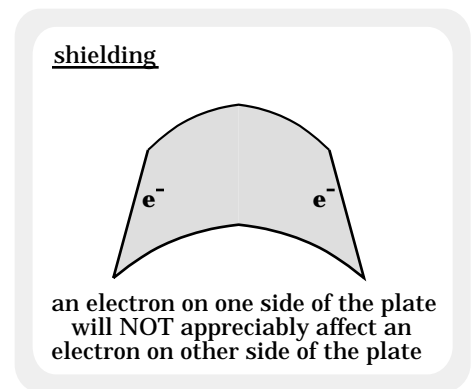


FIGURE 13.18

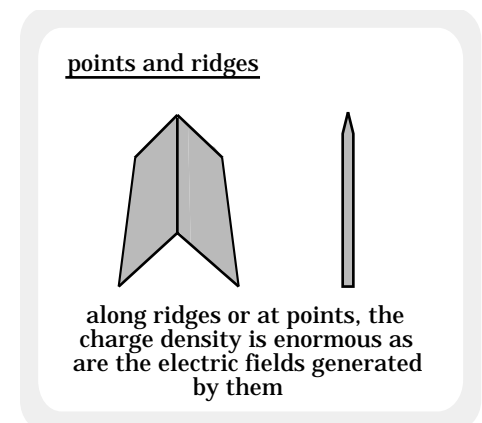


FIGURE 13.19

charged, shielding at the point's radically curved end produces an enormous charge density at the point. That, in turn, produces an enormous electric field in the vicinity of the point. This is the basis for lightning rods.

G.) Electric Dipole Moment:

Note: You will not be tested on the dipole moment material below. Nevertheless, it is presented here because an analogous situation will be observed when we talk about magnetic fields, and because it is customary to do so.

1.) There are many situations in nature when the charge on a body separates to produce what is called an electric dipole. Examples:

a.) Water (H_2O), due to the way the hydrogens and oxygen bond, produces a situation in which the oxygen-side of the molecule is electrically negative while the hydrogen-side is electrically positive.

b.) The two ends of a radio antenna act like a positive/negative charge combination that oscillates back and forth as electromagnetic radiation impinges upon the antenna.

2.) To better understand the concept of the electric dipole, consider two equal but opposite charges of magnitude q separated rigidly a distance $2a$ units apart. If that charge is positioned in an external electric field (see Figure 13.20), they will each feel a torque and attempt to align themselves with the field. How large is that torque, assuming the structure is at an angle θ with the field (again, see Figure 13.20)?

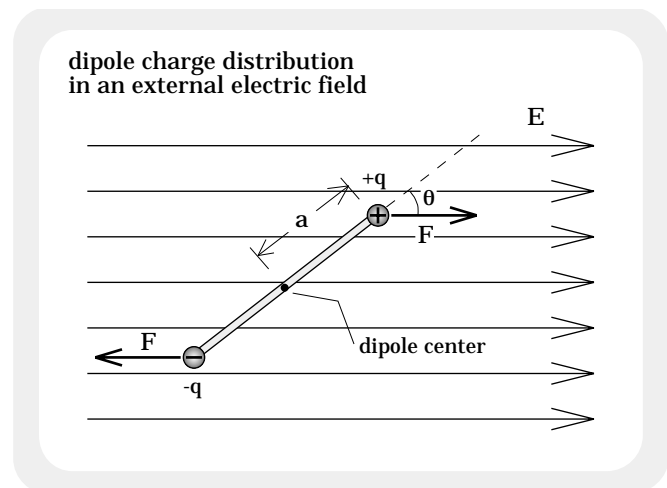


FIGURE 13.20

a.) The force on each charge is qE (this is simply a manipulation of the definition of an electric field). The torques on the charges will be equal and in the same direction (define r for both and use your right-hand rule--the torques are in the same direction). That means the net torque will be twice the torque due to one force.

b.) In this case, the magnitude of the vector r is a , the magnitude of the force F is qE , and the angle between r and F is θ . With all this, we can write the magnitude of our torque calculation as:

$$\begin{aligned}\Gamma &= 2 |r \times F| \\ \Gamma &= 2(a)(qE) \sin \theta \\ &= (2aq)E \sin \theta.\end{aligned}$$

c.) This looks a lot like the magnitude of a cross product executed between a vector whose magnitude is $2aq$ and the electric field vector E (note that the angle between the vector and E is θ).

d.) Being clever, let's define a vector whose direction is along the line between the negative and positive charges (see Figure 13.21) and whose magnitude is $2aq$. If we called this vector the dipole moment p , the torque on the electric dipole when in an electric field becomes:

$$\Gamma = p \times E.$$

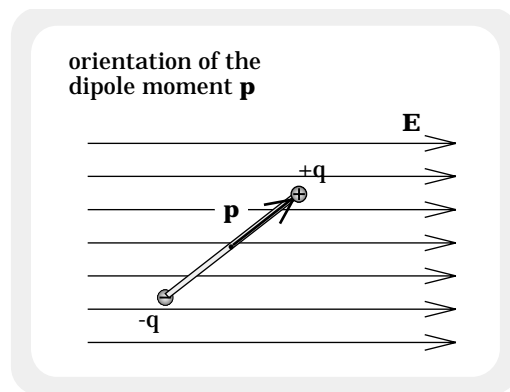


FIGURE 13.21

3.) In addition, in linear systems, potential energy functions are useful whenever we want to calculate the amount of work a force field does as a body moves from one point to another in the field. We can do a similar calculation with one-dimensional, rotational systems by substituting:

$$U(r_2) - U(r_1) = -\int F \cdot dr$$

by

$$U(\theta_2) - U(\theta_1) = -\int \Gamma \cdot d\theta.$$

To do so, consider the following:

a.) Assume an electric dipole is made by some outside force to rotate from an angular position θ_1 to an angular position θ_2 , where $\theta_2 > \theta_1$. How much work does the electric field do during the motion?

b.) The magnitude of the torque applied to the dipole by the electric field when at an arbitrary angle θ is $pE \sin \theta$, but the direction of the

torque in this case (i.e., the situation sketched in Figures 13.20 and 13.21) is negative. How can we tell? Because it attempts to force the dipole clockwise--the direction we have defined as negative in the context of rotational motion. As the rotation itself is counterclockwise, $d\theta$ is positive.

What does this mean? When we do the cross product, the torque and the angular displacement will be in opposite directions, which means the sign of the cross product will be NEGATIVE. Using this information, we can write:

$$\begin{aligned} U(\theta_2) - U(\theta_1) &= -\int_{\theta_1}^{\theta_2} [(pE \sin \theta) d\theta] \\ &= pE [-\cos \theta]_{\theta_1}^{\theta_2} \\ &= -pE(\cos \theta_2 - \cos \theta_1). \end{aligned}$$

c.) This equation implies that the potential energy provided to the dipole by the electric field when the dipole is at an angle θ with the field will be:

$$U(\theta) = -(\mathbf{p})(\mathbf{E}) \cos \theta.$$

d.) This expression is that of a dot product.

e.) Bottom Line: The potential energy wrapped up in an electric dipole situated in an external electric field equals:

$$U(\theta) = -\mathbf{p} \cdot \mathbf{E}.$$

H.) Addendum--Charges, Bonding, Insulators and Conductors:

1.) Like energy, the concept of charge is a very strange duck. That is, we know when charge is present, we know how to generate free charge, we can store charge, and we know how to use charge. What we don't really know is exactly what charge is. In short, when discussing the concept of charge, we are limited to discussing characteristics. The characteristics we will be dealing with are summarized below:

a.) Electrons exhibit the electrical property we associate with negative charge. Electrons are found in the orbital energy shells surrounding the nucleus of atoms. Electrons can exist by themselves outside the atom (in that state, they are called free electrons).

b.) The charge on an electron is called the elementary charge unit. It is equal to 1.6×10^{-19} coulombs in the MKS system, and 1 atomic charge unit in the CGS system.

c.) Protons exhibit the electrical properties we associate with positive charge. Although protons can exist outside atoms, they are generally fixed in an atom's nucleus.

d.) There is the same amount of positive charge on a proton as there is negative charge on an electron.

e.) An object is labeled positively charged if it has more protons in its structure than electrons. As protons are rigidly fixed at the centers of a structure's atoms, this circumstance is achieved by somehow removing electrons from the object leaving a surplus of protons.

f.) An object is labeled negatively charged if it has more electrons in its structure than protons. This circumstance is achieved when electrons are somehow placed on the structure creating a surplus of electrons.

g.) Positive charges attract negative charges. Positive charges repulse positive charges. Negative charges attract positive charges. Negative charges repulse negative charges. In short, likes repulse while opposites attract.

2.) Bonding:

a.) An atom's electrons distribute themselves in well defined energy levels that surround the nucleus. Electrons fill these levels, more or less, from the inside out (that is, from close in to the nucleus to farther out away from the nucleus).

b.) In many cases, an atom will deal with the absorption of energy by boosting one of its electrons into a higher energy level. Assuming that is NOT the case (i.e., assuming the atom is unexcited), the outermost energy level in which electrons are found is called the valence level.

c.) In covalent bonding, an atom whose valence shell is not completely full of electrons will group together with one or a number of other atoms to share valence electrons in an effort to fill its valence shell. This bonding through the sharing of valence electrons is characteristic of insulators.

d.) In ionic bonding, an ion (i.e., an atom that has either lost or gained extra electrons producing a net positive or negative charge on the atom) is attracted to an oppositely charged ion. The most common example of this kind of bonding is table salt--the combination of a positive sodium ion (Na^+) with a negative chlorine ion (Cl^-). Ionic bonding is not something we will deal with much. It has been included here for the sake of completeness.

e.) Metallic bonding is similar to covalent bonding in the sense that there is a sharing of electrons, but whereas valence electrons in covalent bonding are shared by neighboring atoms only, valence electrons in metallic bonding are shared by ALL of the atoms in the structure. That is, valence electrons are not constrained to stay around their original atom--they have the freedom to wander throughout the structure. Conductors (metals) are metallically bonded. Electron mobility is the reason conducting materials heat so easily, and why they can conduct electrical currents.

3.) Electrostatic characteristics of conductors: Rub a rubber rod (this is an insulator) with a wool cloth and the rod will charge. Bring the rod near a small, metallically coated, styrofoam ball (such a ball, metallically coated or not, is called a pith ball) suspended by a string. What happens?

a.) Initially, the ball will be attracted to the rod. Why?

i.) Let's assume the rod's charge is positive (i.e., electrons are rubbed off the rod leaving a preponderance of positive charge). The positively charged rod will attract free to wander valence electrons in the metallically bonded pith ball motivating them to accumulate on the side of the pith ball nearest the rod (see Figure 13.23). As protons are fixed in their respective nuclei, they will not move remaining fixed on the ball.

ii.) With fixed protons on the far side of the pith ball and displaced electrons on the near side, we will have induced an artificial charge polarization.

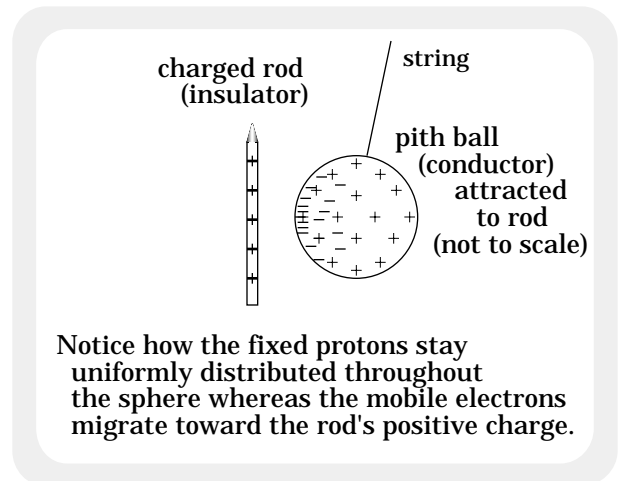


FIGURE 13.23

iii.) Because, on average, the pith ball's electrons will be closer to the rod than are its protons, the attraction between the pith ball's electrons and the rod's protons will be greater than the repulsion between the pith ball's protons and the rod's protons. The net effect will be an attraction of the pith ball to the rod.

b.) After the pith ball touches the rod, the pith ball will swing away due to repulsion. Why?

i.) Electrons will be transferred from the pith ball to the rod when the two touch. Because the rod is covalently bonded, the transferred electrons will stay exactly where the rod and pith ball contacted. That means that the rod's net charge will remain positive (the contact point will be small), but now there will be fewer electrons on the originally neutral pith ball making it also electrically positive. This net positive charge causes repulsion between the pith ball and the positively charged rod, and the pith ball responds by swinging away.

4.) Electrostatic characteristics of insulators: Rub a rubber rod with a wool cloth and the rod will charge. Bring the rod near a pith ball that is not metallically coated (in this case, the uncoated, covalently bonded pith ball will act like the insulator it is). What happens?

a.) As surprising as it may be, the ball will be attracted to the rod just as it was when the pith ball was metallically coated. What is going on here?

i.) The covalently bonded pith ball does not have the kind of electron mobility that would have been the case if it had been a metallically bonded conductor, but it does have electrons that move about in their orbits.

ii.) Using the terribly inaccurate Bohr model of the atom (i.e., electrons moving in perfect circles about a fixed nucleus) to get a visual feel for the situation, the average (mean) position of an orbiting electron is normally at the center of the nucleus. That is, the electron will occupy one side of its orbit as much as it does the other side (remember, electrons move at speeds upwards of 100,000 miles per second), so its average position over time is at the atom's center. This position is on top, so to speak, of the atom's protons, which is why atoms generally appear to be electrically neutral.

iii.) When the positive rod comes close, the pith ball's electrons are attracted and end up spending more time in their orbital motion on the rod's side of the atom. In other words, there is a polarization that occurs inside the atom (see Figure 13.24). Although

this polarization is small (the offset distance must be less than the radius of an atom--approximately a half angstrom, or $.5 \times 10^{-10}$ meters), the attraction of the pith ball's closer electrons is greater than the repulsion of its farther away protons, and the net effect is that the pith ball moves toward the rod.

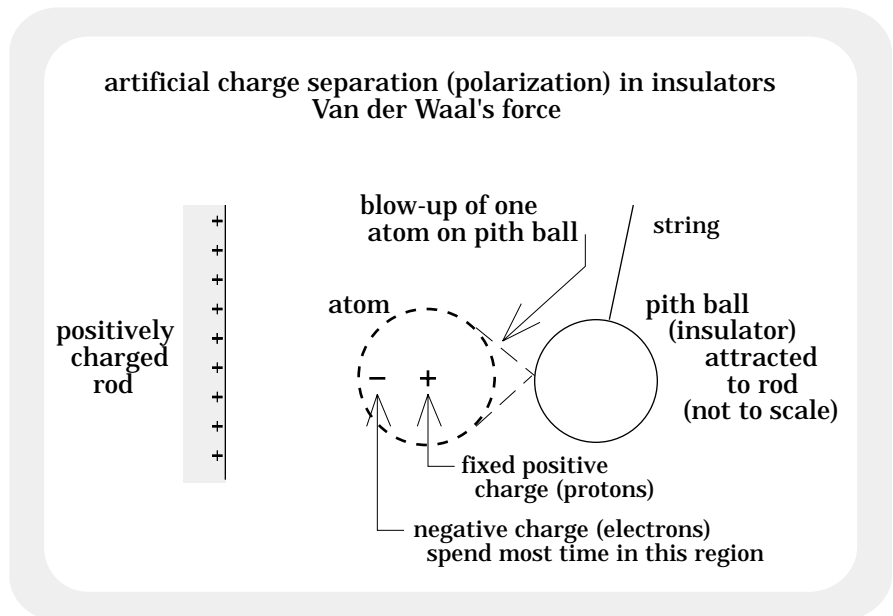


FIGURE 13.24

iv.) This attraction will be strong enough to pull the pith ball to the rod, but it might not be strong enough to rip electrons off the pith ball and onto the rod. If that be the case, the pith ball will hold onto the rod, acting as though there was a slight bond between the two. That bond is called a Van der Waal force. An excellent example of this effect is found when a balloon is rubbed on hair, then placed against the wall. In such an instance, the balloon will stay on the wall until ions in the air can pluck the free charges off the balloon, allowing it to release from the wall and fall to the ground.

v.) If there happens to be enough free charge on the rod, electron transfer will occur just as it did with the metallicly coated pith ball. After touching, the ball will repulse and will move away from the rod.

5.) A path that allows electrons to freely flow onto an object or off of an object is called a GROUND SOURCE. Motors and other electrical devices can have static electricity build up on their chassises. A device that has this problem has a third prong on its power cord. That prong is attached to a wire that is, itself, connected to the chassis of the device. In a wall socket, the hole into which this prong fits (the bottom hole) is connected, quite literally, to a pipe that goes into the ground (often it is a plumbing pipe). This is called a ground connection. If the chassis charges up positively, electrons will be drawn from ground to neutralize the build-up. If the chassis charges negative, that negative charge will drain off via the ground connection. So

knowing all of this, is there some clever way that you can act like a ground to charge a metallic object? The answer is yes.

a.) Bring a charged rod close to the metallic surface you want charged.

b.) Electrons on the metallic surface will rearrange themselves, depending upon whether your rod is positively or negatively charged.

c.) Once the polarization has occurred (this will be instantaneous), touch the side of the metal object that you wish to affect. If you touch the side on which positive charge predominates, you will act as a ground and electrons will flow from you onto the surface. With what is now a preponderance of negative charge, the surface will be negatively charged when the rod is removed. Touch the other side of the surface and electrons will flow off the surface leaving it electrically positive. The kind of charge that is left on the object all depends upon where you touch and ground the object.

QUESTIONS

13.1) Consider the three charges shown in Figure I (see sketch for dimensions).

a.) What is the net electrostatic force on the $-500 \mu\text{C}$ charge?

b.) Where would we have to put the $-500 \mu\text{C}$ charge (assuming it was mobile) if we wanted the net electrical force acting on it to be zero?

c.) Remove all three charges. Place $-q_1$ at $x = 0$ and $+q_2$ at $x = 3$ meters. Assuming the magnitudes of the two charges are the same (I've labeled them differently to delineate between the two), draw a sketch of the electric field versus position graph for the set-up. That is, what will the electric field function look like (in graphical form) for both positive and negative values of x ?

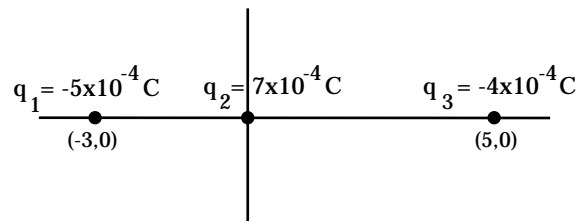


FIGURE I

13.2) A -5×10^{-8} coulomb charge is placed on a particle of mass 2×10^{-4} kg that resides in outer space (i.e., ignore gravity). When the particle is placed in an electric field, a net force of 3×10^{-3} i newtons is felt by the particle.

- What is the magnitude of the electric field?
- What is the direction of the electric field?

13.3) Figure II shows two charges q_1 and q_2 as they are positioned relative to a coordinate axis.

- Using the variables provided, derive an expression for the net electric field as it exists at point $(-a, -b)$.

- Assume $a = .4$ meters, $b = .3$ meters, $q_1 = 7 \mu\text{C}$, and the magnitude of the second charge $q_2 = 5 \mu\text{C}$. Determine a numerical solution for the net electric field at the point $(-.4, -.3)$.

- A $-.012 \mu\text{C}$ charge is placed at position $(-.4, -.3)$. What is the net electrostatic force experienced by the charge?

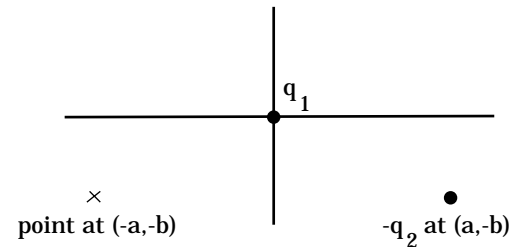


FIGURE II

13.4) A sphere charged to $-.04$ coulombs is found to levitate when placed in an 800 volts/meter electric field (note that a volt/meter is the same as a newton/coulomb).

- In what direction is the electric field?
- What is the mass of the sphere?

13.5) The diameter of a typical hydrogen atom is approximately 10^{-10} meters across. The charge on an electron is the same as the charge on a proton (one elementary charge unit is $e = 1.6 \times 10^{-19}$ coulombs) and the mass of an electron is 9.1×10^{-31} kilograms. If we assume that the electron follows a circular path around the proton:

- What is the electron's velocity magnitude as it "orbits" around the proton (think about the KIND of motion the electron is executing)?
- Determine both the electric force on the electron when in the atom and the gravitational force on an electron when near the earth's surface. Take a ratio of the two and comment.

13.6) Two unequal masses (assume $m_1 = 2m$ and $m_2 = m$) have the same charge q on them. They are suspended by unequal lengths of massless

thread (lengths L_1 and L_2 respectively) from a common point such that when in equilibrium, they share a common horizontal line (i.e., they are on the same level--see Figure III to the right). If mass m_1 makes an angle with the vertical of $\theta_1 = 35^\circ$ while mass m_2 makes an angle of θ_2 :

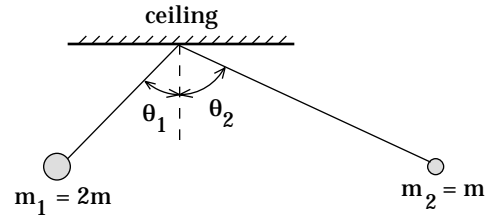


FIGURE III

- Determine θ_2 (try N.S.L. on both masses and see what you can cancel out).
- Assuming mass $m = .03$ kg and $q = 5.5 \times 10^{-10}$ coulombs, what is the distance R between the two masses?

13.7) A cross-section of three long, parallel lines of negative charge is shown in Figure IV to the right. If the linear charge density is the same for all three lines, what do the electric field lines for the system look like?

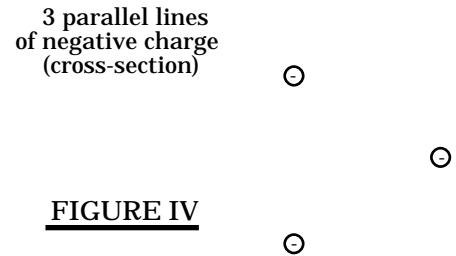


FIGURE IV

13.8) A constant electric field can be produced by putting equal and opposite charges on parallel plates. When done, the electric field in the region between the plates will be constant. Two parallel plates are charged as shown in Figure V to the right (the sketch shows a cross-section of the plates). An electron (mass $m = 9.1 \times 10^{-31}$ kg and charge magnitude $q = 1.6 \times 10^{-19}$ coulombs), having been accelerated to a velocity of 4×10^4 m/s, enters the constant electric field as shown in the sketch.

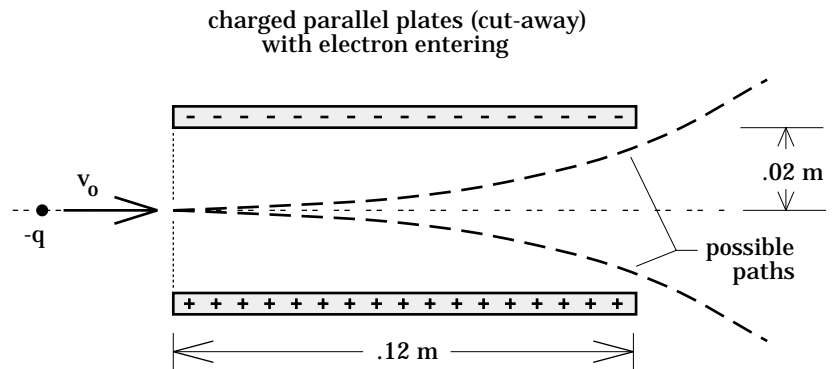


FIGURE V

- On the sketch, what direction is the direction of the electric field?
- On the sketch, in what direction will the electron follow (i.e., which path)?
- If the electric field is too strong, the force on the electron will send it crashing into one of the plates (which plate depends upon which way the field is pointed). Using the lengths shown in the

sketch, what is the largest electric field the system can handle without having it hit the walls?

13.9) A thin, semi-circular rod has a linear charge density of $-k\theta$ on its upper half and is mirrored on its lower half (see sketch). Assume k is a constant with appropriate units.

- a.) What are the units for k ?
- b.) Derive an expression for the electric field at the semi-circle's center.

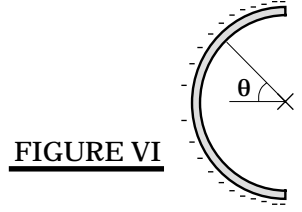


FIGURE VI

13.10) A rod of length L has a uniform charge density of $-Q/L$ on it. Assuming the coordinate axes passes through the center of the rod (see figure VII), determine the electric field at (L,L) . If you can't do the appropriate integrals, at least set up the problem.

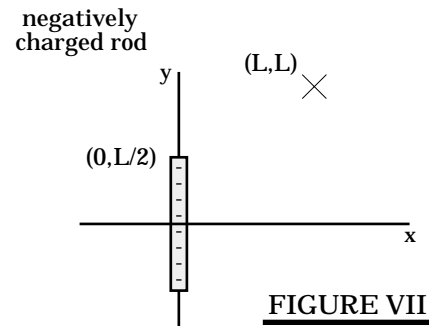


FIGURE VII

13.11) A proton is placed at $x = b$ on the central axis of a charged hoop (total charge $-Q$). If the proton is released, it will accelerate toward the hoop, pass through to the other side, slow, reverse direction, then accelerate again back toward the hoop's center, etc. In other words, it will oscillate back and forth down the axis. Is this motion simple harmonic in nature and, if it is, what is the frequency of the oscillatory motion?

